FORMATION OF CONDENSED OXIDE PARTICLES BY COMBUSTION OF METAL DROPLETS

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The formation of condensed oxide particles in combustion of metal droplets is discussed; it is assumed that the characteristic diffusion time is much less than the characteristic time for the heterogeneous reaction at the condensate particle surfaces, and the structure of the reaction zone is discussed; the size spectrum is derived for the condensed oxide particles. It is found that condensation in the gas has little effect on the droplet combustion rate. Heat needed to evaporate the metal is produced directly at the surface of the drop and the rate-limiting step in the combustion is the diffusion of oxidant to the surface.

A major feature of metal combustion is that the combustion products are particles of condensed material. The theory of metal combustion should include the theory of metal particle formation and give the size spectrum. The formation mechanism and the particle spectrum can be elucidated via the condensation kinetics and the structure of the reaction zone. In constructing a model for metal droplet combustion [1] one extends the model for combustion of a hydrocarbon fuel droplet, which involves the concept of an infinitely narrow flame front, which does not include the structure of the reaction zone.

Research on condensed oxide formation from metals is only in an early stage. One assumes that the oxides of metals such as aluminum and magnesium decompose almost completely on evaporation since they are very refractory compounds, and can thus condense directly from the products by decomposition, i.e., from the vapors of the metal and oxidant, without directly forming the oxide vapor. This assumption was made in [2], and it involves representing the condensation of the oxide as a reaction that begins with the formation of aggregates containing metal and oxygen atoms, which act as nuclei, this continuing at the expense of the metal and oxidant interaction at the surface.

The chemical condensation is accompanied by evaporation and oxide decomposition, the result being an equilibrium with the two rates equal. The oxide formation rate is dependent on deviation from equilibrium, which is defined, as for an ordinary condensation, by the supersaturation $\Omega = (T_e - T)/T_e$, where T is the actual temperature and T_e is the temperature corresponding to chemical equilibrium between the condensed material and the gaseous products.

When a metal droplet burns, one can use as the measure of deviation from equilibrium in the condensation a quantity dependent on the rate of mixing by diffusion between the metal vapor from the droplet and the oxidant from the outside. The more quickly the reactants enter the reaction zone, the greater the excess of condensation over evaporation. It is of interest to consider the case where the diffusion is so rapid that the condensation deviates as far as possible from equilibrium, when the oxide formation rate will not be dependent on the supersaturation and is small relative to the diffusion rates. This situation can occur if the size of the hot metal droplets is sufficiently small, in which case the diffusion fluxes, which are inversely proportional to the droplet diameter, become high; estimates show that this situation can occur for a metal droplet of the size commonly employed in experiments.

The following is the picture of the burning pattern. The metal vapor is oxidized in the surrounding gas, but this has little effect on the metal vapor concentration and oxygen concentration near the droplet;

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the reaction zone in the gas extends far away from the droplet, and has little effect on the penetration of oxidizing gases to the metal surface. The heat needed to evaporate the metal then is provided mainly by the oxidation directly at the droplet.

The calculations then relate to the oxide formation in the gas zone. We do not consider here the formation of oxide at the hot droplet, but it is assumed that the oxide form there does not constitute a continuous film and does not hinder evaporation and oxidation.

Let the condensation rate function be

$$\Phi = k_1 a_f a_{ox}^{l} \rho^{1+l} s \tag{1}$$

where a_f and a_{ox} are the relative volume concentrations of the droplet evaporation products and oxidant, respectively, with ρ the gas density and s the specific surface per unit volume for the condensate particles.

By analogy with the condensation rate we assume that the nucleation rate is also proportional to the condentrations of oxidant and droplet evaporation products raised to some power:

$$dn / dt = k_2 a_f^{p} a_{ox}^{q} \rho^{p+q} \tag{2}$$

where n is the number of nuclei in unit volume.

The oxidation of a metal is a reaction of addition type with a low activation energy, so one assumes that the coefficients of proportionality k_1 and k_2 in (1) and (2) are constant and independent of temperature.

Consider a stationary spherical droplet of radius r_0 in a medium of temperature t_0 containing an oxidant concentration a_{0x}^{0}.

The problem may be simplified via various assumptions. Firstly, let the rate of formation of condensed particles equal the rate represented by the Stefan flux from the droplet surface. Secondly, we assume that the rates of consumption of oxidant and evaporation products in the nuclei are small relative to the condensation rate. Condensation theory indicates that the nucleus size falls rapidly as the supersaturation increases, and the supersaturations are assumed high in the present case, so the nuclei are very small. Thirdly, we assume that reactions (1) and (2) are of second order ($p = q = l = 1$).

We introduce the following dimensionless variables and parameters: $\zeta = r/r_0$ the distance from the droplet surface, $\delta^2 = k_1 \rho r_0^2 s_0/D$ the ratio of the characteristics diffusion time to the characteristic heterogeneous reaction time for the nuclei, $S = s/s_0$ the specific surface of the growing oxide particles, and $\beta/\xi^2 = ur_0/D$ the speed of the Stefan flux. If $\xi = 1$ (at the droplet surface), the latter quantity is equal to $\beta_0 = u_0 r_0/D$, where D is the diffusion coefficient, u is the speed of the Stefan flux, and $s_0 = [4\pi(\rho/\rho_c)^2 k_2 \rho/k_1]^{1/4}$ is the characteristic value for the specific surface of the condensate particles.

We assume that the thermal diffusivities and diffusion coefficients are equal for all substances and independent of temperature and gas composition, in which case the dimensionless equations become as follows: the diffusion equation for the droplet evaporation products:

$$\frac{d}{d\zeta}\left(\beta a_f - \zeta^2 \frac{da_f}{d\zeta}\right) = -\nu_f \delta^2 \zeta^2 a_f a_{ox} S \tag{3}$$

The equation for the Stefan flux speed:

$$d\beta / d\zeta = -\nu \delta^2 \zeta^2 a_f a_{ox} S \tag{4}$$

In (3) we replace subscript f by subscripts ox, p, and i to derive analogous equations for the oxidant, decomposition products, and inert gas. In the equation for the decomposition products from the oxidant the right side should be positive, while in that for the inert gas it should be zero. The parameters ν_f, ν_{ox}, ν_p, and $\nu = \nu_f + \nu_{ox} - \nu_p$ introduced into the equations are the stoichiometric coefficients for the heterogeneous reaction at the surface of the nuclei: $\nu_f[f] + \nu_{ox}[ox] = [c] + \nu_p[p]$, where [f], [ox], [c], [p] are the droplet evaporation products, the oxidant, the condensate, and the oxidant decomposition products.

Consider the boundary conditions needed to solve (3) and (4); at the surface of an evaporating droplet with $\zeta^2 = 1$ these conditions become

$$a_f = a_f^{\circ}, \quad a_{ox} = 0, \quad \left(\beta a_i - \frac{da_i}{d\zeta}\right) = 0, \quad \left(\beta a_p - \frac{da_p}{d\zeta}\right) = -\frac{\nu_p}{\nu_{ox}} \frac{da_{ox}}{d\zeta} \tag{5}$$

This means firstly that the concentration a_f° of the droplet evaporation products at the surface corresponds to the saturation vapor concentration at temperature T_s; secondly, there is a heterogeneous reaction of very high rate at the droplet surface; and thirdly, the droplet surface is impermeable to the decomposition products from the oxidant and inert gas.

The boundary conditions in the surrounding medium for $\xi \to \infty$ become

$$a_{ox} = a_{ox}^{\circ}, \quad a_f = 0, \quad a_p = 0, \quad a_i = (1 - a_{ox}^{\circ}) \tag{6}$$

The right sides of (3) and (4) contain the unknown quantity S; to determine this we consider the equation for the mass change in a single condensate particle and the equation for the nucleation rate in unit volume. We assume that the particles are spherical and that (1) and (2) apply to the condensation and nucleation rates, in which case the equations become: for the particle mass change

$$dg \,/\, d\xi = (36\pi)^{1/3}\rho\delta^2\zeta^2 a_f a_{ox} g^{2/3} \,/\, \rho_c^{2/3}\beta s_0 \tag{7}$$

for the nucleation rate:

$$d\beta n \,/\, d\zeta' = k_2\rho^2 r_0^2 \,(\zeta')^2 a_f\,(\zeta')\; a_{0x}\,(\zeta') \,/\, D \tag{8}$$

We integrate (7) to find the particle diameter as a function of the coordinate $\zeta' = r'/r_0$, at which the nucleus is formed:

$$\varphi = 2\frac{\rho}{\rho_c}\frac{\delta^2}{S_0}\int^{\zeta}\frac{\zeta^2 a_f a_{ox}}{\beta}\,d\zeta \tag{9}$$

We multiply (8) by the area of a particle having the diameter defined by (9) and integrate the expression with respect to ξ' to get

$$\begin{gathered} S = \frac{1}{\beta}\int_0^{\theta_-}(\theta_- - \theta_-')^2\,\beta\,(\theta_-')\,d\theta_-' \\ \theta_- = \delta^2\int_1^{\zeta}\frac{\zeta^2 a_f a_{ox}}{\beta}\,d\zeta, \quad \theta_-' = \delta^2\int_1^{\zeta'}\frac{\zeta^2 a_f a_{ox}}{\beta}\,d\zeta \end{gathered} \tag{10}$$

We substitute (10) into (4) and differentiate with respect to θ_- to get (12)

$$d^4\beta \,/\, d\theta_-^4 = -2\nu\beta \tag{11}$$

We solve this equation subject to the boundary conditions $\theta_- = 0$, $\beta = \beta_0$, $d\beta/d\theta_- = 0$, $d^2\beta/d\theta_-^2 = 0$, $d^3\beta/d\theta_-^3 = 0$ to get (12)

$$\beta = \beta_0 \cos\,[(\nu \,/\, 2)^{1/4}\theta_-]\;\mathrm{ch}\;[(\nu \,/\, 2)^{1/4}\theta_-] \tag{12}$$

This shows that the Stefan flux becomes zero at some point ζ^*, where $\theta_- = \theta_-^* = (2/\nu)^{1/4}\pi/2$.

We then eliminate the right sides from (3) and (4) and integrate the resulting expression with respect to ξ on the basis that the flux of droplet-evaporation products at infinity is zero, which gives

$$\beta a_f - \zeta^2\frac{da_f}{d\zeta} = \frac{\nu_f}{\nu}(\beta - \beta_\infty) \tag{13}$$

where β_∞ is the Stefan flux at infinity.

In the case of a metal droplet burning in water vapor or carbon dioxide ($\nu_p = \nu_{0x}$, $\nu = \nu_f$) the volume flow rate of oxidant decomposition products (H_2 or CO) from the reaction zone equals the volume flow rate of the oxidant to the droplet, hence $\beta_\infty = 0$; if a droplet of metal in oxygen, β_∞ is finite and negative.

To determine how θ_- varies with ξ, we use the following equation derived from (10)

$$d\theta_- \,/\, d\zeta = \delta^2\zeta^2 a_f a_{ox} \,/\, \beta \tag{14}$$

The boundary conditions for (14) is $\xi = 1$, $\theta_- = 0$ by virtue fo the definition of θ_- given by (10).

We subsequently suppose that the characteristic diffusion time r_0^2/D is small relative to the characteristic heterogeneous reaction time at the surface of the nuclei $1/k_1\rho s_0$, which corresponds to $\delta \ll 1$; this will be so if

$$r_0^2 \ll D \,/\, k_1\rho s_0 \tag{15}$$

If $\delta \ll 1$ near a droplet, and hence $\xi\delta \ll 1$, we can neglect the right side in (14), and then the solution subject to the boundary condition will be $\theta_- = 0$; we substitute this solution into (13) and use the boundary condition $\zeta = 1$, $a_f = a_f^{\circ}$, to get the distribution for the droplet evaporation product concentration for $\zeta\delta \ll 1$:

$$a_f = [\nu_f\ (1 - \beta_\infty\ /\ \beta_0)\ /\ \nu] + [a_f^\circ - \nu_f\ (1 - \beta_\infty\ /\ \beta_0)\ /\ \nu]e^{-\beta_0(1/\zeta-1} \tag{16}$$

At large distances from the droplet, where $\zeta\delta \sim 1$, we introduce the new variables

$$\begin{aligned} x &= \zeta\delta\varepsilon\ [(\nu\ /\ 2)^{1/4}\nu a_{ox}^\circ\ /\ \nu_f]^{1/2}, \quad z = a_f\ /\ \beta_0\delta \\ y_- &= (\nu\ /\ 2)^{1/4}\theta_-, \quad \varepsilon = 1\ /\ \zeta^*\ \delta\ [(\nu\ /\ 2)^{1/4}\nu a_{ox}^\circ\ /\ \nu_f]^{1/2} \end{aligned} \tag{17}$$

where ζ^* is the coordinate at which the Stefan flux becomes zero. Then since $a_{0X} = a_{0X}^\circ + O\ (\delta)$, at large distances, we neglect quantities of the order of δ to get

$$\begin{aligned} &\varepsilon^2 x^2 \frac{d}{dx}\left(\frac{\cos(y_-)\,\mathrm{ch}\,(y_-)}{x^2}\frac{dy_-}{dx}\right) = -\left(\cos(y_-)\,\mathrm{ch}\,(y_-) - \frac{\beta_\infty}{\beta_0}\right) \\ &z = \varepsilon^3 \frac{\nu_f}{\nu}\ [(\nu\ /\ 2)^{1/4}\nu a_{ox}^\circ\ /\ \nu_f]^{1/2} \frac{\cos(y_-)\,\mathrm{ch}\,(y_-)}{x^2}\frac{dy_-}{dx} \end{aligned} \tag{18}$$

Although at $\zeta = \zeta^*$ the value of S is infinite, and hence also is the reaction, the consumption of droplet evaporation products in this zone remains finite, so it is possible for gaseous evaporation products from the droplet to penetrate into the region $\zeta < \zeta^*$. We repeat the above arguments to get equations analogous to (18) for the concentration distribution in this region:

$$\begin{aligned} &\varepsilon^2 x^2 \frac{d}{dx}\left(\frac{\cos(y_+)\,\mathrm{ch}\,(y_+)}{x^2}\frac{dy_+}{dx}\right) = -(\cos(y_+)\,\mathrm{ch}(y_+) - 1) \\ &z = \varepsilon^3 \frac{\nu_f}{\nu}\ [(\nu\ /\ 2)^{1/4}\nu a_{ox}^\circ\ /\ \nu f]^{1/2} \frac{\beta_\infty}{\beta_0} \frac{\cos(y_+)\,\mathrm{ch}\,(y_+)}{x^2}\frac{dy_+}{dx} \\ &y_+ = (\nu\ /\ 2)^{1/4}\delta^2 \int\limits_\infty^\zeta \frac{\zeta^2 a_f a_{ox}}{\beta}\,d\zeta \end{aligned} \tag{19}$$

The boundary conditions for (18) and (19) are put as

$$\begin{aligned} &x = 0,\ y = 0;\ x \to \infty,\ y \to 0;\ x = 1,\ y_- = y_+ = \pi\ /\ 2 \\ &\beta_0 \cos(y_-)\,\mathrm{ch}\,(y_-)\frac{dy_-}{dx} = \beta_\infty \cos\,(y_+)\mathrm{ch}(y_+)\frac{dy_+}{dx} \end{aligned} \tag{20}$$

The latter boundary condition shows that there is no concentration discontinuity at $\zeta = \zeta^*$, which can serve to determine the dependence of ε (the coordinate of the point where the Stefan flux becomes zero) on β_∞, which itself is dependent on the initial parameters. We add the diffusion equations in pairs in such a way as to eliminate the right sides and solve the resulting equations subject to the condition $\delta \ll 1$ with $\zeta = 1$ to get

$$a_f^\circ = 1 - \left[1 - \left(1 - \frac{\nu_p}{\nu_{ox}}\right)a_{ox}^\circ\right]e^{-\beta_0} \tag{21}$$

As $a_f \sim \delta$, at large distances, we get from (16) with an accuracy up to quantities of zero order in δ that

$$\frac{\beta_\infty}{\beta_0} = 1 - \frac{\nu}{\nu_f}\left[1 - \left(1 - \frac{\nu_p}{\nu_{ox}}\right)a_{ox}^\circ e^{-\beta_0}\ /\ (1 - e^{-\beta_0})\right] \tag{22}$$

To determine how ε varies with β_∞ and the concentration distribution far from the drop ($\zeta \sim 1/\delta$), we consider (19), restricting consideration to the case $\varepsilon \ll 1$, which is of the major practical importance. We see from (19) that in that case the values of y_+, z, and β change only in a small region around x = 1 of width of the order of ε, with $y_+ = 0$ and z = 0 in the rest of the region of the solution to (19). Near x = 1 we introduce the new coordinate $X = (x - 1)/\varepsilon$ and neglect quantities of the order of ε to get a solution to (19) as

$$\begin{aligned} &\left(\varepsilon \cos(y_+)\,\mathrm{ch}\,(y_+)\frac{dy_+}{dx}\right)^2 = \cos(y_+)\,\mathrm{sh}\,(y_+) + \sin(y_+)\,\mathrm{ch}\,(y_+) - y_+\ /\ 2 - \\ &- [\sin(2y_+) + \mathrm{sh}\,(2y_+)]\ /\ 4 - [\mathrm{sh}\,(2y_+)\cos(2y_+) + \mathrm{ch}\,(2y_+)\sin(2y_+)]\ /\ 8 \end{aligned} \tag{23}$$

Then we use the boundary conditions of (20) to get for x = 1 that

$$\cos(y_-)\,\mathrm{ch}\,(y_-)\frac{dy_-}{dx} = \frac{\beta_\infty}{\beta_0\varepsilon}\left[\mathrm{ch}\left(\frac{\pi}{2}\right) - \pi\ /\ 4 - \frac{1}{8}\,\mathrm{sh}\,\pi\right]^{1/2} \tag{24}$$

Figure 1 and 2 show numerical solutions to (19) and (23); the first shows the relation between ε and $(-\beta_\infty/\beta_0)$, which satisfies all the boundary conditions of (20) and (24). The Stefan flux becomes zero at a finite distance from the droplet surface such that $\varepsilon \approx 0.3$–0.4, not only for combustion in water vapor and carbon dioxide but also in oxygen. The second figure shows how y_- (curve 1), $\beta dy/\beta_0 dx$ (curve 2), and β/β_0 (curve 3) vary with x for combustion in water vapor or carbon dioxide ($\beta_\infty = 0$).

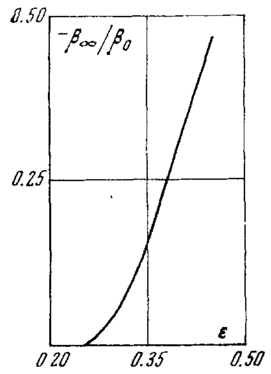

Fig. 1

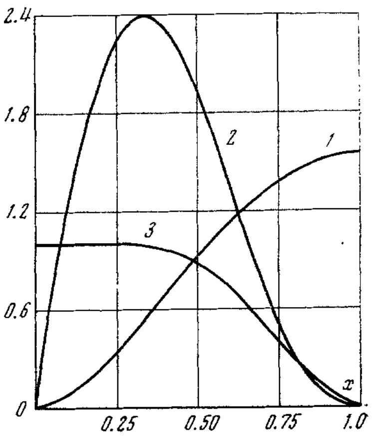

Fig. 2

As the Stefan flux becomes zero at $\zeta = \zeta^*$, the steady-state particle distribution near this point is reached in a time much larger than the characteristic droplet combustion time $2r_0^2/\alpha$; in that sense, the droplet combustion is not even quasistationary, but Fig. 2 shows that in the present case, namely $\varepsilon \ll 1$, the main consumption of evaporation products and the related formation of condensed oxide particles occurs at distance $\zeta \sim 1/\delta$ from the droplet, while the variation in the number and size of the condensate particles near $\zeta = \zeta^*$ is negligibly small. In view of this, the situation is nonstationary only on account of the motion of the oxide particles, not because of formation and growth.

Consider the oxide formation in the combustion of a metal droplet of radius R; we assume that the ratio of r_0^2/D to the characteristic droplet size variation time $2r_0^2/\alpha$ will be less than δ^3 if the law $r_0^2 = R^2 - \alpha t$ applies, and this condition gives

$$r_0^3 > \alpha D^{1/2} / 2\beta_0 (k_1 \rho s_0)^{3/2} \qquad (25)$$

It can be shown that this restriction includes the restriction that the time of flight of an oxide particle formed near the droplet surface out to a distance $\zeta \sim 1/\delta$ is less than the characteristic droplet-size variation time.

We assume that (25) is met and find the particle size distribution for which purpose we differentiate (9) with respect to ζ' and divide (8) by the resulting expression. Then integration over the entire combustion time $\tau = R^2/\alpha$ gives

$$\frac{dN}{d\varphi} = \frac{2}{3} \pi R^3 \frac{\rho}{\rho_c} \frac{2D}{\alpha} \frac{k_2 \rho}{k_1} \beta(\zeta') \qquad (26)$$

This equation goes with (19) in parametric form (parameter ζ') to give the particle size distribution from a burning metal droplet; the maximum oxide particle size in the gas φ_m is found by substituting $\zeta' = 1$ and $\zeta = \zeta^*$ into (9), which gives

$$\varphi_m = \pi \left[(\rho / \rho_c)^2 k_1 / 2\pi\nu k_2 \rho\right]^{1/4} \qquad (27)$$

The result for the particle size spectrum does not constitute the complete spectrum for the droplet combustion products, since a considerable fraction of the oxide may be formed at the droplet surface, and the condensate spectrum calculated for the gas may be substantially modified by coagulation processes, which we do not consider here.

The function of (9), (26), and (27) allows one to determine the mass of oxide formed in the gas by metal droplet combustion; we integrate the product of the distribution and the mass of the oxide particles $\pi\varphi^3\rho_c/6$ with respect to size from 0 to φ_m to get

$$m_0 = \frac{4}{3} \pi R^3 \rho \frac{2D}{\alpha} \frac{(\beta - \beta_\infty)}{\nu} \qquad (28)$$

If there is condensed oxide on the hot metal droplet, the evaporation products are not necessarily metal vapor; when aluminum burns, for example, it can be shown from thermodynamic data [3] that the main equilibrium product from evaporation of condensed metal and oxide will be Al_2O; oxide evaporation along with the metal from the droplet. The oxide formation metal at the droplet surface appears to involve penetration of oxidant, which enters into a heterogeneous reaction with the condensed metal in accordance with $\nu_m[m_c] + \nu_{0x}[ox] = [c] + \nu_p [p]$, where $[m_c]$ is the condensed metal.

When the entire droplet has burned, the mass of oxide is $4\pi R^3 \rho_m / 3\nu_m$, where ρ_m is the density of the metal, so the proportion of oxide formed in the gas phase (reckoned relative to the total mass of oxide) is

$$\delta = \frac{\rho}{\rho_m} \frac{2D}{\alpha} \frac{\nu_m}{\nu} (\beta_0 - \beta_\infty) \qquad (29)$$

We give some numerical evaluations for the basis for the main assumptions made above; first of all we check the assumption that the condensate evaporation rate is small relative to the formation rate, which can be put as $k_e \ll P_0^2 a_f a_{0x}$, for this second-order reaction, where k_e is the constant of the equilibrium between the concentrate and gaseous products. For $Al_2O + O_2 = Al_2O_2$ at 3000° K we have $k_e = 3.66 \cdot 10^{-8}$ atm^2, which shows that the above assumption is correct for all realistic concentrations ($a_{0x} \sim 1$, $a_f \sim \delta$).

The condensation occurs in the presence of excess oxidant, so one supposes that the condensation is proportional to the number of collisions between molecules of the droplet evaporation products and the surface of a condensate particle. Then the rate constant for the condensation is $k_1\rho = wv/4$, where v is the thermal velocity of the gas molecules and ω is the collision efficiency. It has been shown [2] that the mean value is $\omega = 7.5 \cdot 10^{-2}$ for magnesium vapor combustion; the mean speed of gas molecules at 3000° K is about 10^5 cm/sec, so the condensation rate constant $k_1\rho$ is $1.9 \cdot 10^3$ cm/sec.

It is difficult to estimate the nucleation rate constant, since nothing has been published on the question; it has been shown [4] that the maximum oxide particle size produced in the gas by combustion of aluminum and beryllium droplets is about 1 μm. We substitute this value of φ_m into (27) to get the nucleation rate constant $k_2\rho^2$ as $5.9 \cdot 10^{12}$ $cm^{-3} \cdot sec^{-1}$. The following values were used in this calculation: $\nu = 1$, $\rho = 5.1 \cdot 10^{-6}$ mole/cm^3, $\rho_c = 3.6 \cdot 10^{-2}$ mole/cm^3, $\alpha = 8.8 \cdot 10^{-4}$ cm^2/sec, $\rho D = 7 \cdot 10^{-5}$ mole/cm $\cdot$ sec, $\beta = 0.3$.

We substitute these quantities into (15) and get that parameter δ will be small if $r_0 < 3.7 \cdot 10^{-2}$ cm or if the metal particle diameter is less than 740 μm.

We substitute the numerical values into (25) to find ouat the quasistationary approximation applies to a burning metal droplet if $r_0 > 2 \cdot 10^{-3}$ cm or if the diameter of the droplet is larger than 40 μm.

We now consider under what conditions we can neglect the rate of consumption involved in nucleation relative to the condensation rate. In the present case this can be put as $\delta^2 \gg 4\pi\mu^3 k_2 \rho^2 r_0^2 \rho_c / 3\, \rho D$, where μ is the radius of a nucleus. We substitute for the numerical values to get that the assumption is correct if $\mu \ll 2.2 \cdot 10^{-5}$ cm or 0.22 μm.

The the condensed combustion products form in the vapor in a special state in the case of small metal droplets, as the kinetic resistance is large by comparison with the diffusion one. Condensation in the gas over the metal droplet has little effect on the combustion rate; the heat needed to evaporate the metal is produced directly at the droplet surface, and the combustion rate-limiting step is oxidant diffusion.

All these results apply only when the Stefan flux is directed away from the surface of the droplet; to determine α and β_0 we need to consider the heat-balance equation for the droplet surface.

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